A Laser Grating Interferometer

James R. Sterrett,* James C. Emery,†
and John B. Barber‡
NASA Langley Research Center, Hampton, Va.

Nomenclature

b = width of undisturbed fringe as it would appear in the disturbance region with a magnification factor of 1
d = grating constant; width of transparent and opaque intervals
f = focal length of mirror
W = width of light beam
x = distance from grating to source image
θ = angle of diffraction
λ = wavelength of light in a vacuum
ρ = density
S = number of fringes between two conditions
L = length of disturbance
n = index of refraction of light in undisturbed area

Subscripts

1 = disturbed condition
∞ = undisturbed condition

THE monochromaticity, coherence, and small source size of the light from a continuous-wave gas laser allow the use of certain devices in interferometer work which were not possible with other light sources. Thus, a low cost interferometer, which is as easy to build and adjust as a schlieren system, has been developed for fluid mechanics research and is described in this note. This instrument uses a small diffraction grating to combine the light on one side of the usual schlieren beam with that on the other side. Fringes form where these two light beams overlap. The reference beam is immediately adjacent to the beam in which the disturbance is placed. (This under certain conditions is an inherent disadvantage of the present system.) A schematic drawing of the instrument is shown in Fig. 1. Note that the light beam is not divided and then recombined as is necessary for the usual interferometer using other than a laser light source.[1-3]

Divergent light from the laser source is collected by a mirror after part of the light is cut off by a stop as shown in Fig. 1. The width of the light beam W should be approximately

$$W = 2f \tan\theta \quad (1)$$

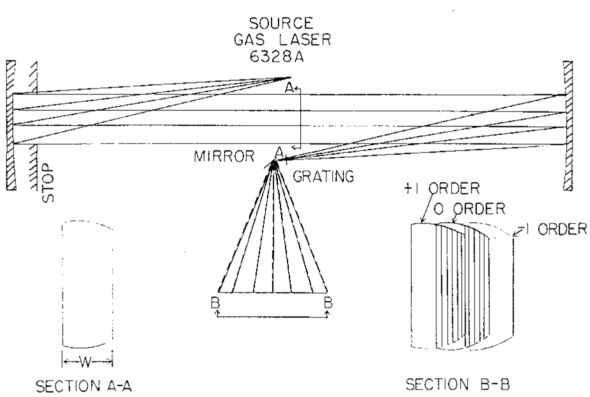

Fig. 1 Laser grating interferometer.

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* Head, 20-Inch Hypersonic Tunnel Section, Aero-Physics Division. Member AIAA.
† Aerospace Engineer, Aero-Physics Division.
‡ Aerospace Engineer, Aero-Physics Division. Associate Member AIAA.

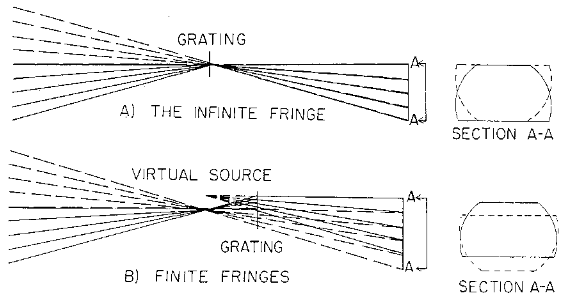


Fig. 2 Light rays near grating.

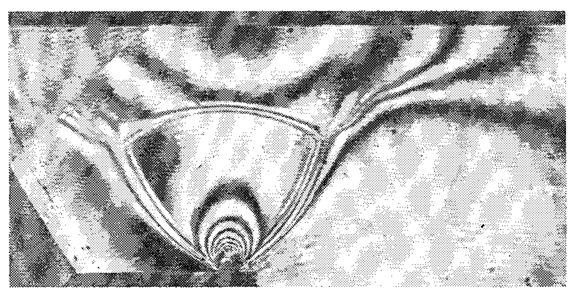
where the value of θ is obtained from the grating equation

$$\theta = \sin^{-1}(\lambda/d) \quad (2)$$

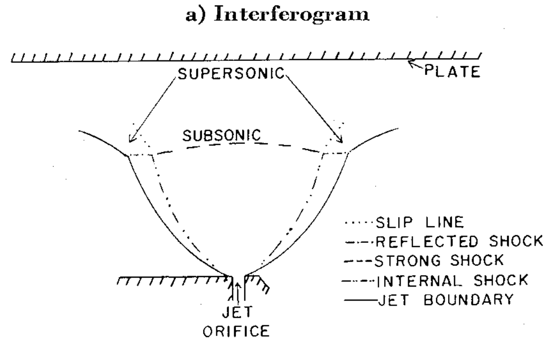
The light rays are refocused by the second mirror on a diffraction grating, which is placed near the image of the light source. The diffraction grating breaks the light beam into a series of diffracted beams or orders. Only three of these orders (−1, 0, and +1) are shown in Sec. B-B of Fig. 1. Fringes form where the diffracted orders overlap. If the grating is located at the light source image, the geometry of the system is such that the rays of the two combining beams are not inclined to one another and the infinite fringe results (see Fig. 2). If the second grating is moved away from the source image, the direction of the rays in both beams remains unchanged, but the distance of any particular ray from the principal optical axial (other than the zero order) has changed (see Fig. 2). Thus the two beams combine at a small angle and finite fringes appear. The number of fringes is thus changed by moving the grating relative to the image of the source. The actual distances in Fig. 2 have been greatly exaggerated. The fringe spacing b can be obtained from the following equation that can be analytically derived from reasoning similar to that given in Ref. 2:

$$b \simeq (fd/x)[1 - (\lambda^2/d^2)]^{1/2} \quad (3)$$

To obtain the infinite fringe pattern, it can be seen from Eq. (3) that the value of x must be very small (the distance b may be assumed to be greater than $W/2$).



a) Interferogram



b) Schematic of jet

Fig. 3. Jet exhausting against a plate.

The equations for determining the density from an interferogram obtained with this type of interferometer are the same as those obtained for the Mach-Zehnder interferometer. For the infinite fringe setting, the fringes resulting from a two-dimensional disturbance represent lines of constant density, the values of which may be obtained from the following general two-dimensional equation[3]:

$$\frac{\rho_1}{\rho_\infty} = \frac{(S_1 - S_\infty)\lambda}{L(n-1)} + 1 \tag{4}$$

If the components of the interferometer are first arranged and adjusted similar to the usual schlieren system, it is only necessary to insert the light stops and grating to obtain fringes. In general, the only other adjustments needed are a translation motion of the grating to obtain the desired fringe spacing and the focusing of the disturbance on the screen.

Figure 3a shows a typical interferogram of a free jet exhausting against a flat plate taken with this instrument. The interferogram was taken with the interferometer adjusted for the infinite fringe pattern. Some of the characteristics of the jet are also sketched in Fig. 3b. A commercial replica transmission grating with 2000 lines per inch was used in the interferometer.

References

[1] Schardin, H., "Theory and application of the Mach-Zehnder interference-refractometer," Univ. of Texas, Defense Research Lab. Rept. UT/DRL-T-3 (1946).

[2] Sterrett, J. R. and Erwin, J. R., "Investigation of a diffraction-grating interferometer for use in aerodynamic research," NACA TN-2827 (November 1952).

[3] Gooderum, P. B., Wood, G. P., and Brevoort, M. J., "Investigation with an interferometer of the turbulent mixing of a free jet," NACA Rept. 963 (1950).

Deflection of a Uniformly Loaded Rectangular Plate with One Pair of Parallel Edges Rigidly Connected to Two Identical Beams

W. W. Smith* and H. H. Pan†
Bell Telephone Laboratories, Inc., Murray Hill, N. J.

Rao discussed the bending of rectangular plates with edge beams but did not give any specific solution.[1] Haskin considered the nonlinear problem of rectangular plates stiffened by beams along two edges,[2] and his work is extensive. However, it is believed that the simple linear solution is still technically valuable. This note presents a series solution for a uniformly loaded rectangular plate with two parallel edges built into two identical, elastic beams of constant cross section and simply supported at the other two edges. This solution embraces the solutions for the following four special cases, namely, 1) uniformly loaded rectangular plate with two parallel edges supported on two identical, elastic beams and the other two simply supported, 2) uniformly loaded rectangular plate with two parallel edges free and the other two simply supported, 3) uniformly loaded rectangular plate simply supported at all of the edges, and 4) uniformly loaded rectangular plate with two parallel edges clamped and the other two simply supported. The solution is obtained as follows: The differential equation for small deflections of a thin plate is

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \tag{1}$$

The required boundary conditions for the problem under consideration are

$$w = 0 \qquad \partial^2 w/\partial x^2 = 0 \quad \text{at } x = 0 \text{ and } a \tag{2}$$

$$EI\left(\frac{\partial^4 w}{\partial x^4}\right)_{y=\pm b/2} = D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right]_{y=\pm b/2} \tag{3}$$

$$-C\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x \partial y}\right)_{y=\pm b/2} = D\left[\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right]_{y=\pm b/2} \tag{4}$$

Equation (2) describes the simply supported condition at $x = 0$ and $x = a$. Equations (3) and (4) describe the rigid attachment of the two edges $y = \pm b/2$ to the elastic beams. In the preceding equations, w is the deflection of the plate; q is the intensity of the uniform load; EI and C are the flexural and torsional rigidity of the plate, respectively; and ν is Poisson's ratio. The dimensions of the plate are $a \times b$. Assume a solution in the form

$$w = \frac{qa^4}{D}\sum_{m=1,3,5\ldots}^{\infty}\left(\frac{4}{m^5\pi^5} + A_m\cosh\frac{m\pi y}{a} + B_m\frac{m\pi y}{a} \times \sinh\frac{m\pi y}{a}\right)\sin\frac{m\pi x}{a} \tag{5}$$

where A_m and B_m are arbitrary constants. It can be verified that the assumed solution satisfies the differential equation and the boundary conditions at $x = 0$ and $x = a$.

Substitution of w from Eq. (5) into Eqs. (3) and (4) results in the following two equations:

$$A_m[(\nu-1)\sinh\alpha_m - m\pi\lambda\cosh\alpha_m] + B_m\{\alpha_m(\nu-1)\cosh\alpha_m + [(\nu+1) - m\pi\lambda\alpha_m]\sinh\alpha_m\} = 4\lambda/m^4\pi^4 \tag{6}$$

$$A_m[m\pi\gamma\sinh\alpha_m + (\nu-1)\cosh\alpha_m] + B_m[(m\pi\gamma\alpha_m - 2)\cosh\alpha_m + m\pi\gamma - \alpha_m(1-\nu)\sinh\alpha_m] = -4\nu/m^4\pi^5 \tag{7}$$

where

$$\lambda = EI/Da \qquad \gamma = C/Da \qquad \alpha_m = m\pi b/2a$$

From Eqs. (6) and (7) a simultaneous solution yields the arbitrary constants A_m and B_m as

$$A_m = (4/m^5\pi^5\Delta)\{[m^2\pi^2\lambda\gamma\alpha_m - 2m\pi\lambda - \alpha_m\nu(1-\nu)]\cosh\alpha_m - [m\pi\lambda\alpha_m - m^2\pi^2\lambda\gamma - \nu(1+\nu)]\sinh\alpha_m\} \tag{8}$$

$$B_m = (4/m^5\pi^5\Delta)\{[-m^2\pi^2\lambda\gamma + \nu(1-\nu)]\sinh\alpha_m + m\pi\lambda\cosh\alpha_m\} \tag{9}$$

where

$$\Delta = [(1-\nu)(\nu+3) - m^2\pi^2\lambda\gamma]\cosh\alpha_m\sinh\alpha_m - \alpha_m(1-\nu)^2 - m^2\pi^2\lambda\gamma\alpha_m - 2m\pi\gamma\sinh^2\alpha_m + 2m\pi\lambda\cosh^2\alpha_m \tag{10}$$

Substituting A_m and B_m into Eq. (5), the desired solution of the problem under consideration is obtained as

$$w = \frac{4qa^4}{D\pi^5}\sum_{m=1,3,5\ldots}^{\infty}\frac{1}{m^5}\left[1 + \frac{1}{\Delta}\left(\{[m^2\pi^2\lambda\gamma\alpha_m - 2m\pi\lambda - \alpha_m\nu(1-\nu)]\cosh\alpha_m - [m\pi\lambda\alpha_m - m^2\pi^2\lambda\gamma - \nu(1+\nu)]\sinh\alpha_m\}\cosh\frac{m\pi y}{a} + \{[-m^2\pi^2\lambda\gamma + \nu(1-\nu)] \times \sinh\alpha_m + m\pi\lambda\cosh\alpha_m\}\frac{m\pi y}{a}\sinh\frac{m\pi y}{a}\right)\right]\sin\frac{m\pi x}{a} \tag{11}$$

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* Member of the Technical Staff.

† Associate Professor, New York University Graduate Center.